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Reviews

Cambridge urban and architectural studies 7: Incidence and symmetry in design and architecture by J A Baglivo, J E Graver; Cambridge University Press, Cambridge, 1983, 306 pages, £27.50 cloth, £9.95 paper (US: $54.50, $15.95)

Incidence and Symmetry in Design and Architecture is written by two professional mathematicians for a wide audience of designers and architects. It had its genesis in teaching material prepared by the Mathematics Department at Syracuse University, New York, for their School of Architecture. It deals with the representation of the properties of spatial objects in a rigorous way. The topological relationships or incidence patterns among spatial objects are represented by graphs and maps. The geometric relationships are confined to symmetrical relationships. These are represented by symmetry groups. Both classes of representations are developed with great pedagogic skill, making them easily accessible to students, practitioners, and researchers in design and architecture. Further, the presentation in terms of clear statements of theorems and selected proofs provides an exciting and stimulating exposure to the rigorous methods of thought required for formal understanding.

The book is divided into two parts. Part one contains four chapters covering the properties of incidence. The spatial properties of an object left invariant by topological transformations in space or in the plane include the incidence relations among different parts of the object. These incidence relations are represented by multigraphs, graphs, and maps. Chapter 1 develops the basic properties of these representations. Chapter 2 describes incidence representations for objects in the plane (and on the surface of a sphere) by planar maps. These maps are used to represent the incidence of spaces in architectural floor plans. Properties of planar maps derived using Euler's formula are given. Those maps which are regular and semiregular are shown (apart from a few explicit exceptions) to correspond to the regular and semiregular polyhedra. Chapter 3 gives further applications of graph theory including bracing a rectangular lattice-like structure to make it rigid. This application illustrates a particularly elegant use of bipartite graphs to represent bracings and to characterize rigidity. Optimal route design is accomplished by dividing regions which the routes traverse into square subregions and representing the routes from subregion to subregion by the edges of a graph. The notion of distance and mean distance in a graph is defined and theorems to determine mean distance in various corridor configurations in buildings are stated and proved. Last, in chapter 3, triangulations are used to represent the organization graphs of the spaces in an architectural floor plan. Chapter 4 presents a useful introduction to the geometry of surfaces and the maps on these surfaces. Tessellations of the plane and compact surfaces both receive elegant if brief treatment.

Part 2 also contains four chapters and deals with the representation of the spatial properties of objects which remain invariant under isometries (distance-preserving transformations). These are the symmetries of the objects and are represented by the isometries leaving the objects invariant. These isometries form a group. The properties of groups are developed when required throughout this part of the book. Chapter 5 describes the isometries of the plane and the basic group theory required to describe them. The fundamental result that every isometry of the plane is either a reflection, rotation, translation, or glide reflection is stated and proved. The group of isometries of the plane is examined in terms of the compositions of the different types of plane isometries. Chapter 6 classifies planar patterns in terms of their symmetry groups, namely the subgroups of the isometries of the plane which leave the patterns invariant. The development concentrates on discrete subgroups of the group of isometries (a group of isometries is discrete, if for any point the individual application of all the isometries in the group yields a discrete set of points—the orbit of the point). Plane patterns are considered equivalent, if their discrete symmetry groups are isomorphic and isomorphism preserves the type of transformation, namely takes reflections to reflections, rotations to rotations, translations to translations, and glide reflections to glide reflections. The remainder of chapter 6 enumerates the classes of discrete symmetry groups of the plane according to the number of nontrivial translations they contain: circular groups (no translations), frieze groups (translations in parallel directions), and wallpaper groups (two translations in different directions).

These classes are discussed in detail. Chapter 7 enumerates the possible space isometries in three dimensions and classifies the discrete space groups in terms of spatial elements left invariant by the group. A discrete space group is: a spherical group if it leaves a point invariant, a rod group if it leaves a line invariant (and is not a spherical group), a layer group if it leaves a plane invariant (and is not a spherical or rod group) and is a crystallographic group otherwise. The rod groups and layer groups are described in detail. Chapter 8 brings together the studies of incidence and symmetry in an enumeration of incidence patterns under various symmetries. These symmetries are often combinatorial and not related directly to the isometries considered in the previous three chapters. The symmetries of graphs are discussed, including the use of a Cayley diagram to represent finite groups. Burnside's lemma considers a group acting on a set and relates the number of elements in a finite group, the number of orbits, and the number of elements in the set left invariant by each group element. This 'lemma' is proved and is an essential tool in enumerating incidence patterns where patterns can be transformed into one another by the action of a group. For example, two labelled graphs which can be transformed into one another by a permutation of the labels are isomorphic under the action of the group of permutations. Thus from a knowledge of the number of labelled graphs of a certain type the nonisomorphic graphs can be enumerated using Burnside's lemma. Applications are given which include the enumeration of nonisomorphic subgraphs of a complete graph and 'minimal' rectangulations of a rectangle, under the action of the symmetry group of the rectangle (the dihedral group D_2). Last the number of triangulations of a polygon, corresponding to the organization graphs of architectural floor plans with all spaces adjacent to the 'outside' space, are enumerated.

The material is presented in a self-contained manner with a close integration between text and the exercises. The book may be used profitably in many ways. For individual study, the numerous exercises and experiments are all well explained and should present no real difficulty for those who have understood the main text. The exercises are not collections of esoteric mathematical results but genuinely develop extensions and practical applications of results established in the text. For a course of tutored study, the book provides a well-ordered exposition of results in graph theory and symmetry as applied to spatial objects which form the basis for any course in this area. Further applications, exercises, and illustrations can easily be incorporated into such a course, because the fundamental ideas are clearly developed. Finally, and by no means least, this book may be used as a reference in the best sense. It is not merely a compilation of results, but a logical development of fundamental ideas and the ways they can be used. The book brings together many ideas relevant for design and architecture, which are scattered throughout the mathematical and design literature.

The work is well produced in generous format and is copiously illustrated with over three hundred line figures. It can be strongly recommended to students, practitioners, and researchers in architecture and design who wish to understand the properties of their designs and the possibilities for new designs. The authors are to be congratulated on an excellent and extremely useful book.

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A search for common ground edited by P Gould, G Olsson; Pion, London, 1982, 277 pages, £12.00 (US: $25.00)

This is a book to be read; a book to be lived; a book to be thought about. It is an extravagant book—a luxury. If in practice its aims are perhaps not fully fulfilled, does that matter? The written word is dead—would you ever write the same thing twice?—but, can it (the phrase) inspire? If so, read on . . .

A Search for Common Ground is the outcome of a conference of fifteen geographers held at the Villa Serbelloni in Italy in the summer of 1980. Twelve papers are presented; Gould provides the prologue, Olsson the epilogue. Nine nations are represented. Let us begin at the beginning, with Gould: the aim. "It was with such a desire to open clarifying discourse between geographers of different persuasions, that the authors in this volume (and one or two others) met in common concern" (page 2). And so the review must be a discourse; the dialectic is the response. And the aim continued: "Our hope now is that these essays will encourage others to take up some of the